Capital income taxation and inherited taste in an endogenous fertility model: a note

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Abstract

Many developed countries have been experiencing declining fertility rates in recent years. This study develops an overlapping generations model that incorporates fertility choices and standard-of-living aspirations. Furthermore, we introduce public investment financed by capital income taxes. We show that an increase in capital income tax promotes not only economic growth, but also fertility in the long run.

JEL classification: H20, J13, and O10.

Keywords: Capital income tax, Endogenous growth, Endogenous fertility

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1 Introduction

Many developed countries are currently facing the serious issue of declining fertility. According to Becker and Barro (1988), increased childcare costs with economic growth reduce fertility rates. In contrast, Kaneko et al. (2016) and Gori and Michetti (2016) indicate that aspirations for better standard-of-living, which represent inherited tastes from previous generations, constitute another causal factor for declining fertility in developed countries. If households desire to maintain a higher living standard than that of previous generations, they may lose financial flexibility to raise their children.

Based on Kaneko et al. (2016) and Gori and Michetti (2016), this study provides a solution for the decline in fertility in developed countries. Our study varies from the above-mentioned two studies in two main areas. First, in line with Barro (1990), we assume that public investment is an engine of endogenous growth. Second, we introduce capital income tax[3] to finance public investment.

A decline in fertility has been reported in several developed countries. Maintaining per-capita growth while improving fertility is a serious economic issue for the governments of developed countries. Our study constructs a simple overlapping generation model that incorporates bequeathed tastes. It is shown that an increase in capital income tax promotes not only economic growth, but also fertility in the long run if bequeathed tastes exist.

The remainder of this paper is organized as follows. Section 2 describes our proposed model. Finally, section 3 concludes the study.

2 Model

2.1 Households

Consider a standard overlapping generations economy at discrete time-periods. The identical households experience two stages: young and old. When households are young, they are endowed with one unit of labor, which is supplied to the labor market inelastically. We assume full employment. Households earn wage income, which is divided between consumption, savings, and childcare during the young stage. When households become old, they retire and consume their savings. Following de La Croix

[3] Uhlig and Yanagawa (1996) demonstrated that increasing capital income tax can promote economic growth in an overlapping generations model.

(1996)[4], Kaneko et al. (2016), and Gori and Michetti (2016), the utility function is as follows:

$$\log(c_t - \rho h_t) + \beta \log d_{t+1} + \gamma \log n_t. \tag{1}$$

c_t denotes consumption in the young stage, h_t, the aspirations inherited from parents, $\beta < 1$ is the discount factor, d_{t+1} is the consumption in the old stage, $0 < \gamma < 1$ is the taste for children and n_t is the number of children. Note that $0 < \rho < 1$ is the intensity for aspirations. Denote N_t as the population size born in period t. Then, the evolution of the population size is represented by $N_{t+1} = n_t N_t$. The budget constraints for households are as follows:

$$c_t + s_t + \sigma w_t n_t = w_t, \tag{2}$$

$$d_{t+1} = [1 + (1 - \tau) r_{t+1}] s_t, \tag{3}$$

s_t denotes savings, w_t, the wage, τ, the capital income tax and r_{t+1}, the interest rate. Note that $\sigma w_t n_t$ is the total expenditure on final goods for child care. We assume $0 < \sigma < 1$ and it portrays that the child rearing cost is a fraction of working income, in line with Fanti and Gori (2014). The optimal allocation is as follows:

$$c_t = \frac{w_t + (\gamma + \beta)\rho h_t}{(1 + \beta + \gamma)}, \tag{4}$$

$$n_t = \frac{\gamma(w_t - \rho h_t)}{(1 + \beta + \gamma)\sigma w_t}, \tag{5}$$

$$\frac{s_t}{n_t} = \frac{\beta \sigma w_t}{\gamma}. \tag{6}$$

From Equation (5), $\partial n_t / \partial h_t < 0$. If households want a higher standard of living than their parents, fertility declines. Conversely, we obtain $\partial n_t / \partial w_t > 0$ from Equation (5). An increase in wages improves fertility in case of bequeathed tastes.

Following de La Croix (1996), Kaneko et al. (2016), and Gori and Michetti (2016), we assume that:

$$h_t = c_{t-1}. \tag{7}$$

2.2 Firms

Identical firms employ labor and capital to provide final goods in a competitive market. The production technology is denoted by:

$$Y_t = A K_t^{\alpha} (E_t N_t)^{1-\alpha}, \; A > 0, \; 0 < \alpha < 1, \tag{8}$$

Y_t is the total output; K_t and N_t denote, respectively, the total capital stocks and total labor inputs; and E_t is the labor efficiency. Assuming zero depreciation, the factor demand is given by,

[4] de La Croix (1996) did not contain fertility choice.

$$w_t = (1-\alpha)Ak_t^{\alpha}E_t^{1-\alpha}. \tag{9}$$

$$r_t = \alpha A k_t^{\alpha-1}E_t^{1-\alpha}. \tag{10}$$

Where $k_t \equiv K_t/N_t$ denotes the per capita capital. From equations (8), (9) and (10), we obtain $Y_t = w_t N_t + r_t K_t$.

This study introduces the endogenous growth generated by public investment, as shown by Barro (1990). Following Kalaitzidakis and Kalyvitis (2004), E_t can be expressed as

$$E_t = \frac{K_t^{\delta}G_t^{1-\delta}}{N_t}, 0 < \delta < 1. \tag{11}$$

Where G_t denotes the public capital. If $\delta \to 0$, the labor efficiency can be rewritten as G_t/N_t as assumed by Maebayashi (2013). Note that if we assume $E_t = G_t/N_t$, our analytical results do not change qualitatively.

2.5 Government

The government collects capital income tax to finance public investment under a balanced budget. The government budget constraints are as follows:

$$\tau r_t s_{t-1} N_{t-1} = G_t. \tag{12}$$

The left-hand side of this equation indicates the tax revenue from capital income tax. In contrast, the right-hand side of the equation captures public expenditure.

2.6 Equilibrium

In equilibrium, the following equation[5] holds.

$$k_{t+1} = \frac{s_t}{n_t}. \tag{13}$$

Substituting Equations (6), (10), and (20) into Equation (22), we obtain

$$k_{t+1} = \frac{\beta\sigma\lambda(\alpha\tau)^{\mu}k_t}{\gamma}. \tag{14}$$

Where $\lambda \equiv (1-\alpha)A^{\overline{\frac{1}{1-(1-\delta)(1-\alpha)}}}$ and $\mu \equiv \frac{(1-\delta)(1-\alpha)}{1-(1-\delta)(1-\alpha)} > 0$. Denoting $g_t \equiv k_{t+1}/k_t$ as the growth rate, we obtain the following constant growth rate.

$$g = \frac{\beta\sigma\lambda(\alpha\tau)^{\mu}}{\gamma}. \tag{15}$$

The right side of equation (15) indicates the cost of childcare. From this equation, we obtain,

[5] See Appendix for the derivation of Equation (13).

$$\frac{dg}{d\tau} = \frac{\beta\sigma\varepsilon\lambda\alpha^{\mu}\tau^{\mu-1}}{\gamma} > 0. \tag{16}$$

An increase in capital income tax promotes economic growth. This is because an increase in capital income tax increases public investment, which, in turn, raises not only wages, but also child-rearing costs. Higher child-rearing costs temporarily reduce fertility, thereby promoting capital accumulation, as shown by Fanti and Gori (2009)[6]. Next, we analyze the dynamics of the economy. From Equations (4), (7), (9), (11), and (12), we obtain the following dynamics of aspiration:

$$h_{t+1} = \frac{\lambda(\alpha\tau)^{\mu}k_t + (\gamma+\beta)\rho h_t}{(1+\beta+\gamma)}. \tag{17}$$

Suppose $x_t \equiv h_t/k_t$ is the ratio of aspirations to per-capita capital. From Equations. (14) and (17), the dynamics of x_t are expressed as follows:

$$x_{t+1} = \frac{h_{t+1}}{k_{t+1}} = \frac{\gamma}{\beta\sigma(1+\beta+\gamma)}\left[1 + \frac{(\gamma+\beta)\rho x_t}{\lambda(\alpha\tau)^{\mu}}\right]. \tag{18}$$

Figure 1 depicts the dynamics of x_t.

[Figure 1: Dynamics of x_t]

If $x_{t+1} = x_t = x$ holds, a unique and stable steady state exists. In this case, the growth rates of h_t and k_t are equal in the long term. From Equation (18), we obtain x as follows:

$$x = \frac{1}{\frac{\beta\sigma(1+\beta+\gamma)}{\gamma} - \frac{(\gamma+\beta)\rho}{\lambda(\alpha\tau)^{\mu}}}. \tag{19}$$

To ensure stable steady state, we impose the following condition.

$$\frac{\beta\sigma(1+\beta+\gamma)}{\gamma} - \frac{(\gamma+\beta)\rho}{\lambda(\alpha\tau)^{\mu}} > 0. \tag{20}$$

If this condition is violated, households will expend all wage income consumption in the young stage to maintain living standards during that period. In other words, it is impossible to achieve both, savings and childcare. In this case, the economy is unsustainable. Hence, we use Equation (20) to focus on the meaningful steady state in the remainder of this study. Differentiating Equation (19) with respect to τ, we derive:

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[6] Fanti and Gori (2009) demonstrated that child tax promotes capital accumulation, which leads to the improvements of fertility in the long run.

$$\frac{dx}{d\tau} = -\frac{(\gamma+\beta)\rho\mu\tau^{-\mu-1}}{\lambda\alpha^{\mu}\left[\frac{\beta\sigma(1+\beta+\gamma)}{\gamma} - \frac{(\gamma+\beta)\rho}{\lambda(\alpha\tau)^{\mu}}\right]^2} < 0. \quad (21)$$

Higher capital income increases public capital, thereby increasing working income. An increase in the working income has the following two effects. First, it increases the aspirations from Equation (17), causing an increase in x. Second, it increases child-rearing costs[7] and temporarily reduces fertility. From Equation (14), a decline in fertility promotes per-capita capital accumulation, which leads to a decrease in x. Because the second effect dominates the first, a rise in capital income tax reduces the ratio of aspirations to per capita capital in the long run. Figure 2 shows the impact of capital income tax on x.

[Figure 2: Impact of capital income tax on the steady state]

We now analyze how capital income tax affects fertility in the long run. Using Equations (5), (9), (11), and (12), one can obtain the following long-run fertility rate in a generic form:

$$n = n[w(\tau), x(\tau)]. \quad (22)$$

By differentiating this equation with τ, we obtain:

$$\frac{dn}{d\tau} = \overbrace{\underbrace{\frac{\partial n}{\partial w}}_{+}\underbrace{\frac{\partial w}{\partial \tau}}_{+}}^{+} + \overbrace{\underbrace{\frac{\partial n}{\partial x}}_{-}\underbrace{\frac{\partial x}{\partial \tau}}_{-}}^{+}. \quad (23)$$

An increase in capital income tax increases public capital, which has two positive effects on fertility, as described below. First, an increase in wages, along with an increase in public capital, directly improves fertility. Second, increasing public capital reduces the long-run ratio of aspirations to per-capita capital in Equation (19), which improves fertility. Consequently, higher capital income tax improves fertility in the long term. By substituting Equations (9), (11), and (12) into Equation (5), we derive the following long-run fertility rate:

$$n = \frac{\gamma}{(1+\beta+\gamma)\sigma}\left[1 - \frac{\rho x}{\lambda(\alpha\tau)^{\mu}}\right],$$

$$= \frac{\gamma}{(1+\beta+\gamma)\sigma}\left\{1 - \frac{\rho}{\left[\frac{\beta\sigma(1+\beta+\gamma)\lambda(\alpha\tau)^{\mu}}{\gamma} - (\gamma+\beta)\rho\right]}\right\}. \quad (24)$$

From this equation, we observe that the long-run fertility rate is constant. We

[7] Recall that the childcare costs rise with a rise in wage as explained by Equation (2)

assume a sufficiently large value of A to ensure that $n > 0$. Differentiating this equation with respect to τ, we obtain:

$$\frac{dn}{d\tau} = \frac{\beta\rho\lambda\mu\alpha^{\mu}\tau^{\mu-1}}{\left[\frac{\beta\sigma(1+\beta+\gamma)\lambda(\alpha\tau)^{\mu}}{\gamma} - (\gamma+\beta)\rho\right]^2} > 0. \tag{25}$$

Above all, we arrive at the following proposition.

Proposition 1

If inherited tastes exist, then an increase in capital income tax promotes economic growth and fertility in the long run.

Table 1 provides a numerical example and investigates the long-run impact of capital income tax on growth and fertility rates. This numerical example is composed of eight parameters, that is $\alpha, \beta, \gamma, \delta, \rho, \sigma, A$ and τ. We assume that the rate of capital income tax lies between 0.1 and 0.5. The rest of the parameters are set as follows: $\alpha = 0.33, \beta = 0.25, \gamma = 0.35, \delta = 0.5, \rho = 0.1, \sigma = 0.1$ and $A = 25$.

[Table 1: Impact of capital income tax on the long run growth rate, ratio of aspirations to per-capita capital and fertility rate]

Table 1 shows that raising capital income tax promotes economic growth and long-term fertility.

3 Conclusions

In developed countries, inherited tastes lead to a decline in fertility. Our study constructs a simple overlapping generations model that incorporates fertility choices, inherited tastes, and public capital. Capital income taxation contributes to improvements in economic growth and fertility if inherited tastes exist.

Appendix

In this Appendix, we derive Equation (13). $C_t \equiv (c_t + \sigma w_t n_t)N_t + d_t N_{t-1}$ and I_t are, respectively, the total expenditure for final goods and total investment at period t. Because we assume zero depreciation, the aggregate capital stock in period $t+1$ is given by $K_{t+1} = I_t + K_t$. The cleaning conditions in the goods market are described as follows:

$$Y_t = C_t + I_t + G_t = (c_t + \sigma w_t n_t)N_t + d_t N_{t-1} + K_{t+1} - K_t + G_t. \quad \text{(A.1)}$$

As denoted above, $Y_t = w_t N_t + r_t K_t$ holds. Using Equations (2), (3) and (12), Equation (A.1) can be rewritten as follows:

$$w_t N_t + r_t K_t = (w_t - s_t)N_t + (1 + r_t)s_{t-1}N_{t-1} + K_{t+1} - K_t. \quad \text{(A.2)}$$

From this equation, one can obtain:

$$(1 + r_t)(K_t - s_{t-1}N_{t-1}) = K_{t+1} - s_t N_t. \quad \text{(A.3)}$$

Recall that the evolution of the population size is given by $N_{t+1} = n_t N_t$. To satisfy Equation (A.3) for any period t, Equation (13) has to hold.

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Figure 1

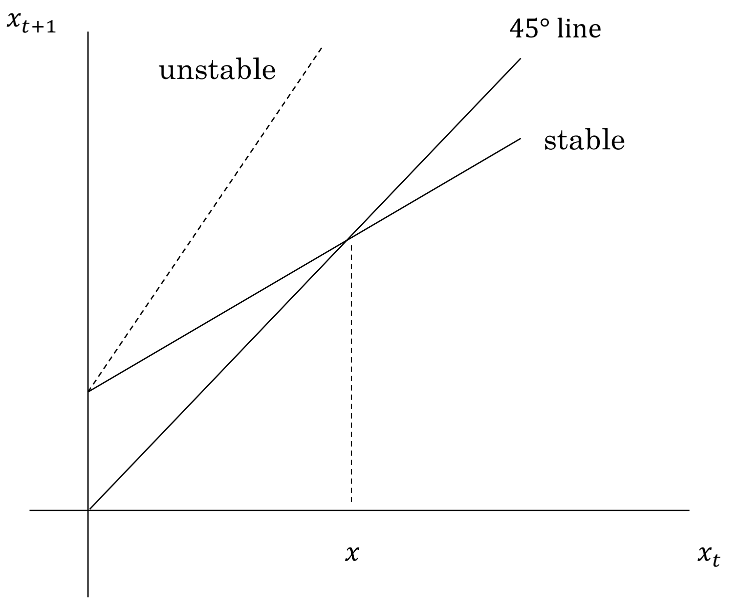

Figure 2

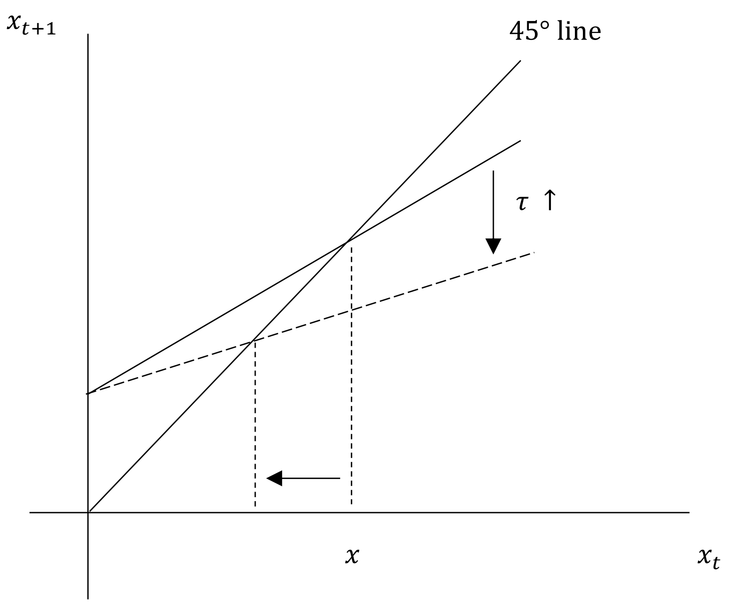

Table 1

τ	0.1	0.2	0.3	0.4	0.5
g	1.086	1.540	1.889	2.183	2.443
x	9.063	8.968	8.927	8.903	8.886
n	2.057	2.096	2.114	2.124	2.131