

Notes on Graphic Methods of Statistical Analysis

TADASHI (Gan) WATANABE

I. Introduction

The following paper is an evaluation of Mr. Louis Bean's "Graphic Methods of Curvilinear Correlation", which contains his first papers published in the Journal of the American Statistical Association, 1929, Wilfred Malenbaum & John Black's criticisms published in the Quarterly Journal of Economics in 1937, with comments by Mordecai Ezekiel, and concluding remarks by Malenbaum. This book was published in 1969 as one of the Reprints of Economic Classics by Kelly. The writer read also M. Ezekiel & K. A. Fox's "Methods of Correlation and Regression Analysis", copyrighted in 1930. Its third edition was published by Wiley & Sons in 1959. In the latter it is written that Bean is the inventor of the graphic method and survived about 50 years,

F. n. 1 The present writer's articles analysing prices of U. S. agricultural products (wheat, corn and cotton in 1941 and potato in 1949) might be compatible, but he regrets that they were published only in Japanese and not cared by the above authorities.

2. Late dr. M. Aramata introduced the Bean method in Japan just after the publication. See Hokeikai Ronso, 1930.

3 The writer graduated the Agricultural Economics Department of Hokkaido University in 1917, worked in the Hokkaido Agricultural Experiment Station for 10 years, and appointed as professor in alma mater, published a paper on "Analytical Economic Comparisons of Experimental Farms Directed by the Hokkaido Agricultural Experiment Station", 1944, "A. Defence of Rice Cultivation with Some Reference to the Climatic Cycles", 1950, and "An Economic Cycle Explanation Beyond the Cobweb

II. Similarities and Differences of Bean and Mordecai Methods

Mr. Bean starts from the separation of groups of similarity in more scattered correlation diagram, and drawing straight lines in each groups, thus of gaining parallel straight lines, gets the mean regression line. Then the dispersions of points from the regression line are connected in each correlation diagram getting the curvilinear forms of multiple correlation.

In contrast to mr. Bean's method, mr. Ezekiel fits the straight lines or curves directly on the scatter (correlation) diagram. So-called correlations are calculated between those straight lines or curves, which are ultimately the flat or curved surfaces in two or three dimentions. The points on the lines and surfaces would show the coordinates, so the volumes surrounded by the surfaces would be the final aim. Thus there should be the integrations of distances from the points to straight line or curves and then to the volumes contained.

The dispersions of points from those lines, surfaces and volume are the ultimate problem.

III. Transformation of Plates

The simplest example may be the hits of gun shots aiming at a point of target circle. Co-central circles are drawn on the target plate, in which the hits scatter, probably dense near and coars farther from, the center.

Then the target plate may be seen obliquely as the circles transformed into ellipses and at last become into a straight line.

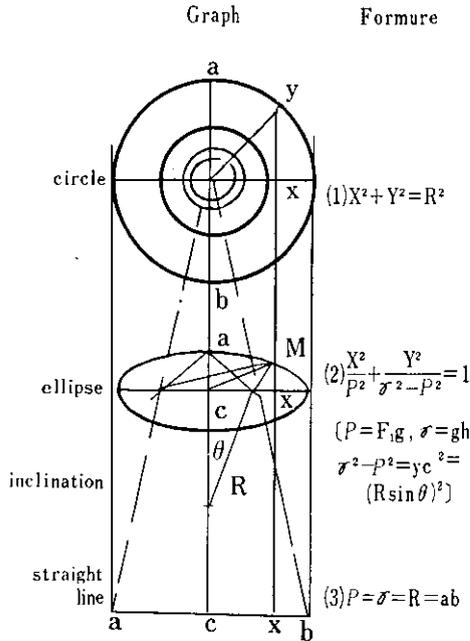
The scatters of hits would be in the probability or normal distribution. The inclinations of the target might be considered as the changes of measuring unit on the ordinate. The distributions of hits are drawn

Theorem", 1967, the latter two in English.

Recent jobs have been the teaching economics, statistics and mathematics in private colleges in Hokkaido.

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ordinate as the probability curve on the abscissa line.



The use of ellipse to reveal the relations was achieved by Augustus Brabaise in early 19 century (*Analyse mathématique sur les probabilités des erreurs de situation d'un point.*) Miss Helen Walker wrote in her *Studies in the History of Statistical Methods, 1929*, "Brabais was unfortunately more concerned with the area than with any relationship between variables... (but) he suggests... when the coordinates are transformed so that the areas of ellipse coincide with the coordinate axes, the equation reduces to the form :

$$Z = \frac{1}{\pi} \sqrt{hxhy} \cdot e^{-(hx^2 + hy^2)}$$

h being $1/2 \sigma^2$

Transformation could be

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$$\log Z = \log \frac{(h_x h_y)}{\pi} - (h_x x^2 + h_y y^2)$$

Thus the formula has the form of dispersion form like Charlier distribution, which has elliptic form. For the writer it seems that the plate may be inclined and circle will be seen as ellipse, which has two foci instead of central point of the former. The distance of between foci extends to the length of the straight line similar to diameter of the circle.

The area of a circle $X^2 + Y^2 = r^2$ is πr^2 , and that of an ellipse derived from the equation $\frac{X^2}{a^2} + \frac{Y^2}{b^2} = 1$

$$\text{is } |D| = \frac{4b}{a} \int_0^a \sqrt{a^2 - X^2} dX = \frac{4b}{a} \left(\frac{a^2}{2} \arcsin \frac{X}{a} + \frac{1}{2} [X \sqrt{a^2 - X^2}] \right) = \pi ab$$

Thus the points distributed around the center could be revealed as ellipse. All points (of hits) are contained in those areas and line, in cases of

- 1) circle $Y^2 + X^2 = R^2$
- 2) ellipse $Y = R \sin \theta, X = R \cos \theta$

$$\sqrt{Y^2 + (x - \rho)^2} + \sqrt{Y^2 + (x - \rho)^2} = (2r)^2$$

$$\text{or } 2 \sqrt{\rho^2 + R \sin \theta} = r^2$$

- 3) Straight line in which only Xs are pointed, but Ys can be drawn as a perspective curve of probability distribution Ae^{-hx} which is derived from binomial

$$(A + B)^n = A^n \left(1 + \frac{B}{A} \right)^n = A^n e^{\frac{B}{A} n}$$

denoting $\frac{B}{A} n = -h^2$, results $A^n e^{-h^2}$

- 4) Thus the curve shows the cutted spindle.

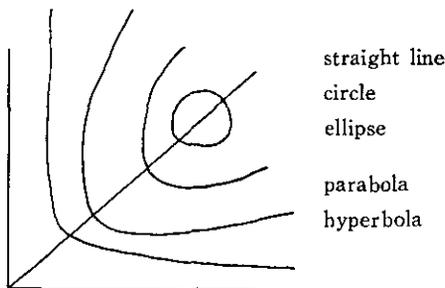
IV. Correlations

Mrs Bean and Mordecai both devised the drawing regressions by their methods. So called correlations may consist between two regressions and by compounding more regressions. In all the dispersions should be diminished. But it should be noted that all points have each dispersions (probably caused with the minor disturbance factors), and a regression, which is the connection of points accompany those borders.

Mr. Ezekiel wrote the philosophy of correlation in the last chapter of his above mentioned book. For the writer it seems that the regressions and the correlations should be more fully examined, for in later time the names, variances and covariances, take place in statistics.

Tomas Baise had in the late 18 century published the idea of variance X , Y and covariance $X \times Y$.

In ordinary synthesis of relationship or a two dimation (second order) can be $ax^2 + by^2 + cxy + gx + hy + j = 0$ in which the first two relate to an ellipse and cxy to a hyperbola and the latter three to a straight line. Thus the general pheature of dispersions on the two dimation plate contain all such relations. Those can be perceived without any integration or differenciation ideas.



V. A Note on Analytic Method for Price Fluctuations

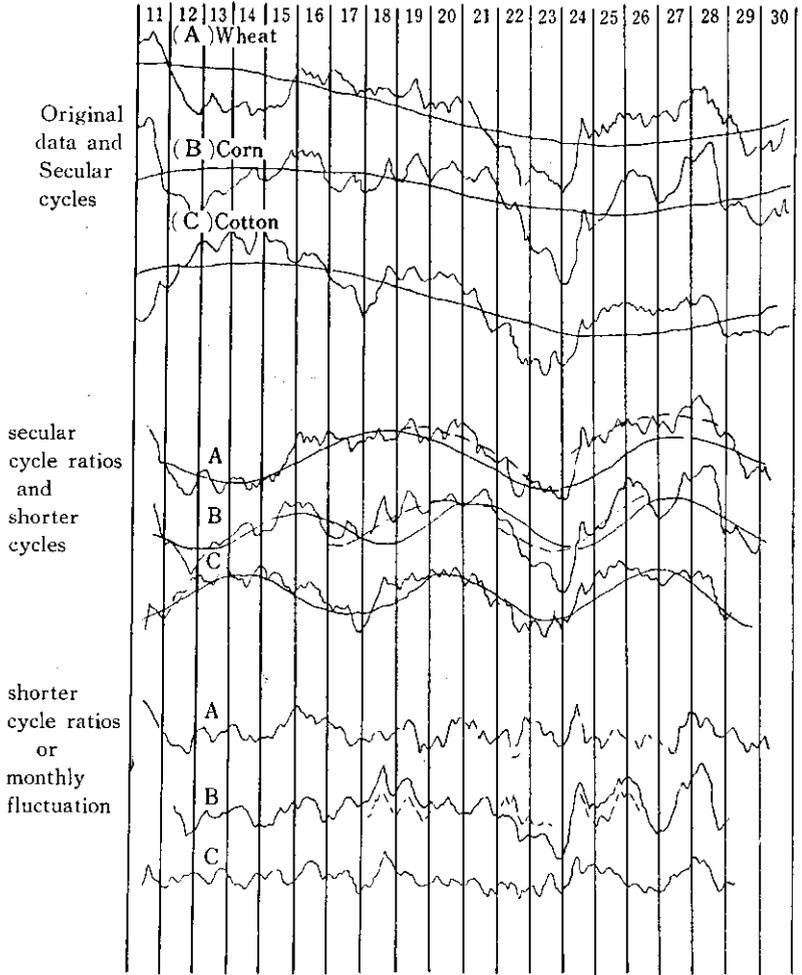
The writer received a letter from Mr. L. Bean who had been once the head of Bureau of Economics, Department of Agriculture, U. S. A. His "Graphic Methods of Curvilinear Correlation" was published by Kelly recently (1969) as one of the Reprints of Economic Classics, with some comments and rejoinders. Mr. M. Ezekiel also published the third edition of his 1931 "Methods of Correlation and Regression Analysis" in 1959. Mr. Ezekiel wrote about Mr. Bean as the inventor of the simplified graphic method in late 1920's. Thus the method survived for thirty years.

The writer commenced the new method in 1930's, in which the logarithmic scales were introduced. He regrets that the publications were done in Japanese, so cared not by foreign authorities while Japanese students did not understand.

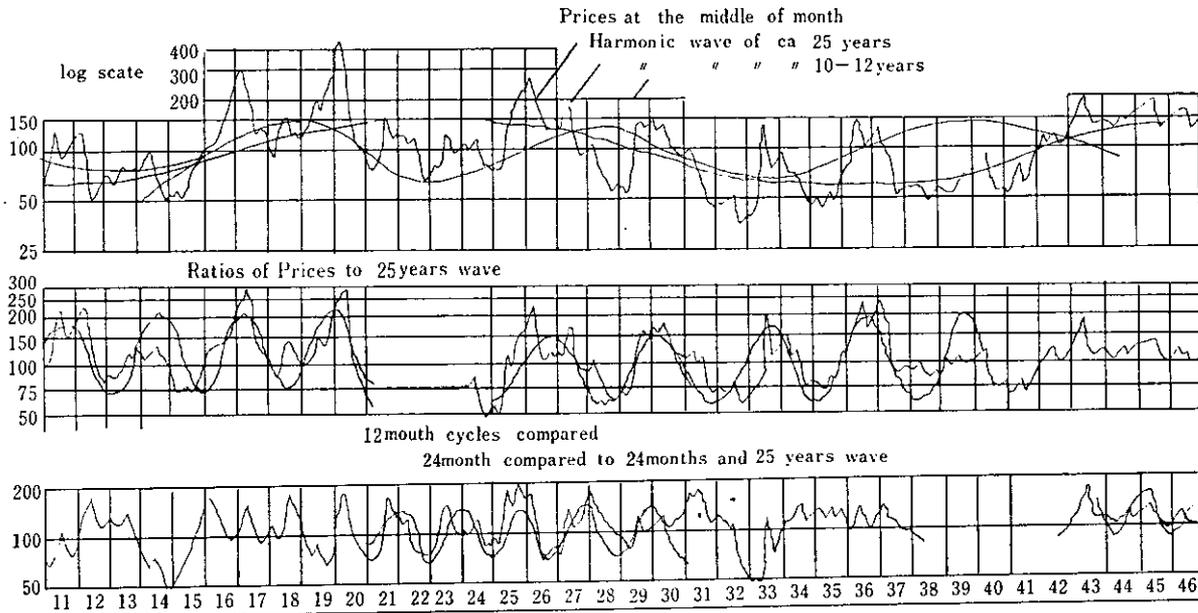
The writer wrote "An Economic Cycle Explanation beyond the Cobweb Theorem" in 1966 (Hokusei-Ronshu, No. 4) in which he confessed the idea was derived from late Dr. J. A. Schumpeter but the analytic methods to use only a compass are writers invention. This method is convenient to count percentages on logarithmic section papers.

This time only the graphs of the analysis and the similarities between them and the sunspots cycles are added in graphs.

Analyses of Monthly Estimated Prices U. S. A.



Analyses of U.S.A. Potato Farm Prices (cent per bushel)



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Those yearly fluctuations can be compared to the sunspot cycles in area and latitude on the solar circle.

Latitudes of sunspot appearance might change the influences on the earth, though the spots number decrease in time of neighbouring to the sun's equator.

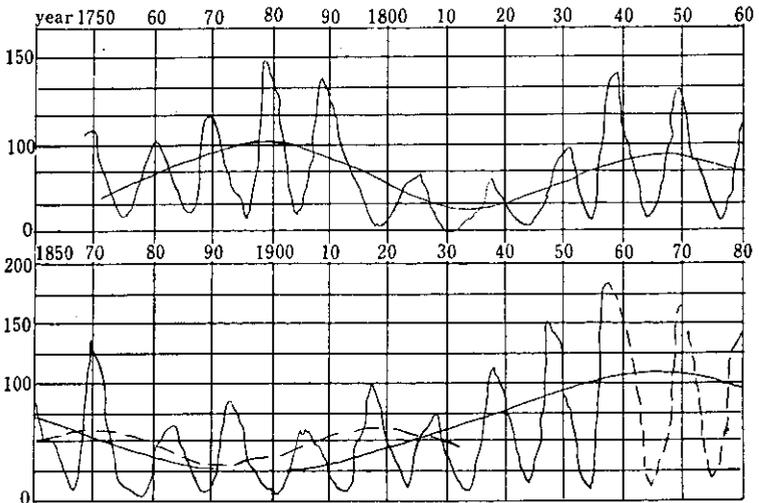


So the writer calculated the probable influences in adding area and latitude measures by weighting the latter.

Those cycles might be compared to the sunspot cycles. They show long wave of 60-100, middle waves of about 11 years, and probably some short waves. or complexes $(3.3)^3 = 118.658$, $(3.3)^2 = 35.97$, $(3.3)^1 = 3.3$ and $(3.3)^0 = 1$.

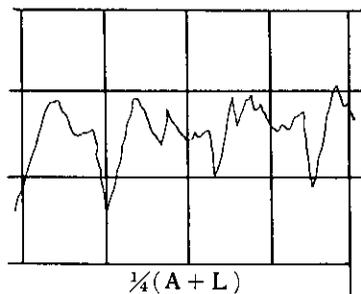
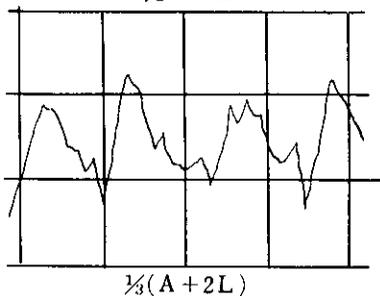
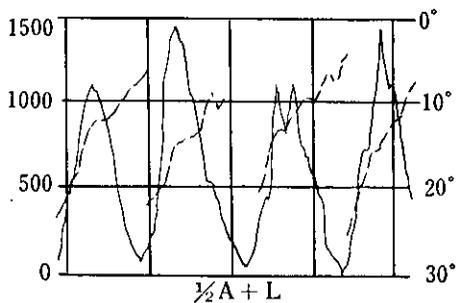
They might be compared to the Kondratief, Juglar (or Jevons) and Kitchin of J. A. Schumpeter. and annual season cycles.

Sunspots fluctuation



SUNSPOTS

AREA (I) LATITUDE



year 1880 1890 1900 1910 1920

(I) Encyclopaedia Britannica 1953, vol. 6, p. 455 & vol. 21, p. 560.
The additions are writes idea

2. How the unequability of international economic development is related to the domestic unequability.
3. Form an estimate of the law.

Accounting and Auditing in School Corporations

Kenzo NAKAGAWA

No generally accepted accounting principles have been established in "school corporation accounting," nor have they in other types of nonprofit corporations.

However, auditing by certified public accountants is now to be enforced on school corporations, since the public subsidies to private schools by the national and local governments are being extended to include a part of the salaries of the staff and the total amount is expected to increase.

The auditing in such a case will be based on the "Standard of School Corporation Accounting" authored by a committee organized under the auspices of the Ministry of Education.

The purpose of this paper is to point out some problems which seem questionable in the "Standard," and to critically review the issues especially related to legal auditing.

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